Activity 1.1.1 Keeping the Peace

The Peacekeeping Problem

Read the following problem. Your team will have approximately 20 minutes to explore the problem and develop the "best" possible solution.

> The United Nations (UN) needs to move Hummers units (one Hummer unit has 5 Hummer vehicles) and Apache helicopters for ground and air support to a region in the world for peacekeeping purposes. A ship is used to transport these vehicles and can accommodate a total equipment weight of 150 tons. Each Hummer unit weighs 15 tons and each Apache helicopter weighs 6 tons. Each Hummer unit takes up an area of 1,200 square feet and each Apache helicopter take up an area of 1,800 square feet. The ship has a total of 25,200 square feet of area to use for transport. What combination of Hummer units and Apache helicopters will yield a maximum force? Force is defined as the total number of Hummer units and Apache helicopters.

Do your work on the back of this page. After working in your group, submit the following information and answer the following questions.

A. Group Member Names:

B. Number of Hummer units to transport:

C. Number of Apache helicopters to transport:

D. Is the total weight (in tons) of the Hummer units and Apache helicopters less than or equal to 150 tons? Show your calculation.

E. Is the total area (in square feet) for the number of Hummer units and Apache helicopters less than or equal to 25200 square feet? Show your calculation.

F. Why do you think you have the best possible combination of vehicles that maximizes the UN's force?

Introduction to Linear Programming

We have just examined each group's "best" solution to the UN peacekeeping problem and came to an agreement on the overall "best" solution. How do we know that this solution is truly the best? Today we will learn about a mathematical technique called *linear programming* which allows us to find the best (optimal) solution to problems like the peacekeeping problem.

This activity will focus on the first two steps in solving a linear programming problem: *define variables for unknowns in the problem*, and *use defined variables to write constraints for the problem*.

Step 1: Define variables for the unknowns in the problem

In the peacekeeping problem, there are two items that need to be transported: Hummer units and Apache helicopters.

- Let x represent the number of Hummer units to be transported.
- Let y represent the number of Apache helicopters to be transported.

Step 2: Use defined variables to write constraints for the problem

The ship used to transport these vehicles can accommodate a total equipment weight of 150 tons. Each Hummer unit weighs 15 tons and each Apache helicopter weighs 6 tons. We can create the following weight-constraint inequality:

$$15x + 6y \leq 150$$

Let's explore how various combinations of number of Hummer units and number of Apache helicopters *satisfy* or *fail to satisfy* the weight-constraint inequality. An ordered pair (x, y) satisfies the inequality if, when it's a and y values are substituted , it makes the inequality a true statement.

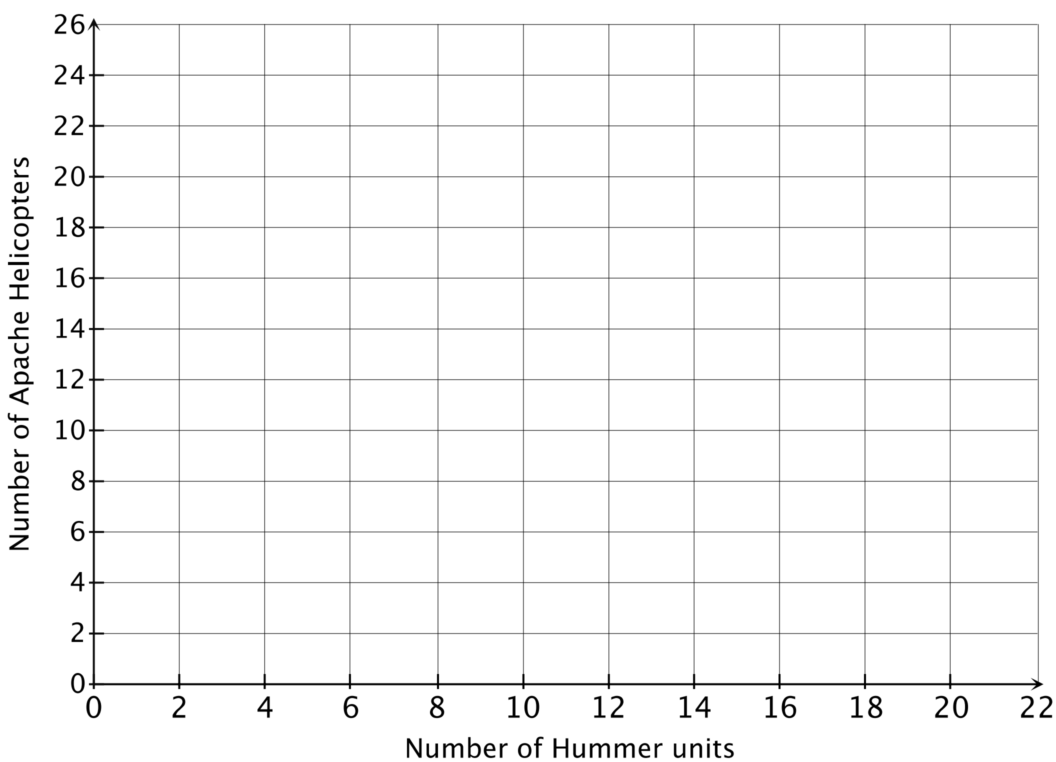
1. Fill in the table below by finding two ordered pairs (x, y) that satisfy the weight-constraint inequality, and one ordered pair (x, y) that does not satisfy the weight-constraint inequality.

x	y	$15x + 6y \leq 150$	Satisfy the Constraint?
4	8	$15(4) + 6(8) \leq 150$	Yes
10	20	$15(10) + 6(20) \leq 150$	No

2. The solutions to the peacekeeping problem must satisfy all conditions in the problem. One common sense condition is that the number of Hummers units and number of Apache helicopters cannot be negative. Write inequalities that state (a) that the number of units of Hummers cannot be negative and (b) that the number of Apache helicopters cannot be negative.

3. Plot the points in your table on the coordinate plane below. Color green the ordered pairs that satisfy the weight-constraint inequality, and color red the ordered pairs that do not satisfy the weight-constraint inequality. *Note*: Only the first quadrant of the coordinate plane is provided since we only want to focus on ordered pairs with non-negative values.

Graph of Weight-Constraint Inequality



4. Graph the equation $15x + 6y = 150$ on the coordinate plane above. This line is called a *boundary line*.

5. Make a conjecture about the location – with respect to the boundary line – of points that satisfy the weight-constraint inequality.

Group Activity

Your teacher will provide each group a transparency. Create a graph of the weight-constraint inequality $15x + 6y \leq 150$ on the transparency. Plot the boundary line and all the ordered pairs that your group explored in Question 1. Color green the ordered pairs that satisfy the weight-constraint inequality, and color red the ordered pairs that do not satisfy the weight-constraint inequality.

Answer the following questions:

6. You may not have been able to make a conjecture about the location – with respect to the boundary line – of points that satisfy the weight-constraint inequality based solely on your individual points. Make a conjecture about the location of points that satisfy the inequality based on all the points from your group.

7. Is it necessary to find a lot of solutions of $15x + 6y \leq 150$ in order to graph this inequality? How many solutions do we need to find, assuming we have plotted the boundary line?

8. There is a second constraint in the peacekeeping problem that is based on the total area of the ship and the area of each Hummer unit and Apache helicopter. The problem stated that "Each Hummer unit uses an area of 1,200 square feet and each helicopter uses an area of 1,800 square feet. The ship has a total of 25,200 square feet of area to use for transport".

   Write an inequality that represents the area constraints in the problem.